



Munich Personal RePEc Archive

## **On the evolution of prize perceptions in contests**

Boudreau, James W. and Shunda, Nicholas

University of Texas-Pan American, University of Redlands

25. August 2010

Online at <http://mpra.ub.uni-muenchen.de/24640/>  
MPRA Paper No. 24640, posted 26. August 2010 / 22:25

# On the Evolution of Prize Perceptions in Contests

James W. Boudreau\*

University of Texas-Pan American

and

Nicholas Shunda†

University of Redlands

August 25, 2010

## Abstract

We apply an indirect evolutionary approach to players' perceived prize valuations in contests. Evolution in finite populations leads to preferences that overstate the prize's material value and induce overexpenditure. We establish an equivalence between the behavior evolutionarily stable preferences induce and evolutionarily stable strategies.

*Journal of Economic Literature Classification Codes:* C72, C73, D72, D74

*Keywords:* Contest, Prize valuation, Preference evolution, Evolutionarily stable preferences

---

\*University of Texas-Pan American, Department of Economics and Finance, 1201 West University Drive, Edinburg, TX, 78539-2999. Tel.: 956-381-2830. E-mail: boudreaujw@utpa.edu

†Corresponding Author. University of Redlands, Department of Economics, 1200 East Colton Avenue, PO Box 3080, Redlands, CA 92373-0999. Tel.: 909-748-8569. E-mail: nicholas\_shunda@redlands.edu.

# 1 Introduction

In a contest, players compete by making irrecoverable investments of expenditure or effort to influence their probability of winning a prize. The most-studied forms in the contest theory literature are “Tullock” contests where each player’s winning probability is her expenditure’s share of total expenditure (Tullock, 1980) and all-pay auctions where the player making the largest expenditure wins for certain (Baye et al., 1996). Contest theory applies to many topics, including rent-seeking and lobbying for political favors, conflict, litigation, R&D competition and patent races, sporting competition, and charitable fundraising.<sup>1</sup> Much of the contest theory literature studies the symmetric case where players have the same valuation that matches exactly the material value of the prize at stake.<sup>2</sup> Players, however, may have asymmetric prize values that also may differ from a prize’s material value because of, for example, nonpecuniary utility of winning, emotions, or interdependent preferences.<sup>3</sup>

Understanding behavior (and the preferences driving that behavior) in contests is important not only because of the many applications of contest theory outlined above, but also because expenditures often exceed risk-neutral Nash equilibrium levels in experimental contests.<sup>4</sup> We address this overexpenditure observation by drawing upon the indirect evolutionary approach of Güth and Yaari (1992) in which players optimize given preferences and preferences evolve according to the fitness they induce. We study the evolution of players’ perceived prize valuations in pairwise contests and identify cases where evolutionarily stable preferences do and do not overstate the material value of the prize and, thus, where induced behavior does and does not lead to overexpenditure. We also establish an equivalence between the behavior evolutionarily stable preferences induce and evolutionarily stable strategies.

This paper contributes to a growing literature on evolutionary approaches to contest theory. Leininger (2003) and Hehenkamp et al. (2004) study evolutionarily stable strategies

---

<sup>1</sup>On these applications of contest theory, see Tullock (1980) and Baye et al. (1993) on rent-seeking and lobbying, Hirshleifer (1995) and Garfinkel and Skaperdas (2007) and the references therein on conflict, Farmer and Pecorino (1999) and Baye et al. (2005) on litigation, Baye and Hoppe (2003) on innovation tournaments and patent races, Szymanski (2003) on sporting competition, and Morgan (2000) and Goeree et al. (2005) on the use of contests for charitable fundraising. Nitzan (1994) and Corchón (2007) provide surveys of contest theory more generally.

<sup>2</sup>For exceptions, see Hillman and Riley (1989), Baik (1994), Baye et al. (1996), Nti (1999), Fang (2002), and Stein (2002) on asymmetric valuations.

<sup>3</sup>Sheremeta (2010) provides evidence of nonpecuniary utility of winning and Herrmann and Orzen (2008) provide evidence of spiteful preferences in experimental contests.

<sup>4</sup>See, for example, Öncüler and Croson (2005), Hörisch and Kirchkamp (2010), Morgan et al. (2010), and the literature cited in these papers for an overview. Interestingly, even Potters et al. (1998), who correct for design flaws in earlier contest experiments, find evidence of overexpenditure in their “Tullock” contests.

in contests and show that they exceed risk-neutral Nash equilibrium levels when evolution occurs in finite populations. Among the papers studying the evolution of preferences in contests are Wärneryd (2002) on risk attitudes, Eaton and Eswaran (2003) and Leininger (2009) on interdependent preferences, Konrad (2004) and Schmidt (2009) on altruism and envy, and Mohlin (2010) on conflict-reducing norms.

## 2 Model and Results

Each period, players from a population of size  $N \geq 2$  match pairwise to compete in a simultaneous-move one-shot contest with a prize of material value  $v > 0$ . Players compete by submitting irrecoverable expenditures  $x_i \geq 0$  to influence their probability of winning, which is given by

$$p_i(x_1, x_2) := \begin{cases} \frac{x_i}{x_1 + x_2} & \text{if } x_1 + x_2 > 0 \\ \frac{1}{2} & \text{otherwise} \end{cases} \quad (1)$$

for  $i = 1, 2$ ; see Skaperdas (1996) and Kooreman and Schoonbeek (1997) for axiomatizations of this well-known contest success function from Tullock (1980).

Preferences are endowed with a perceived value of the prize,  $v_i > 0$ ,  $i = 1, 2$ , which may differ from  $v$ . Evolution operates upon the perceived prize valuations in this model. Each player selects  $x_i$  to maximize expected utility, given by

$$EU_i(x_1, x_2) := \frac{x_i}{x_1 + x_2} v_i - x_i \quad (2)$$

for  $i = 1, 2$ . The following lemma characterizes equilibrium expenditure behavior when an arbitrary pair of players are matched to compete in a contest.

**Lemma 1.** *The unique Nash equilibrium expenditures when players with prize valuations  $v_1$  and  $v_2$  compete in a contest are given by*

$$x_i^*(v_1, v_2) = \frac{v_i^2 v_j}{(v_1 + v_2)^2} \quad (3)$$

for  $i, j = 1, 2$ ,  $i \neq j$ .

*Proof.* The proof is identical to proofs in Nti (1999) and Fang (2002) and hence is omitted.  $\square$

From (3), it is clear that the player with the larger perceived prize valuation makes a larger expenditure and has a higher probability of winning in equilibrium. Of course, the

fitness a player's behavior induces governs the preferences selected as evolutionarily stable. In following the literature on the indirect evolution of preferences—e.g., Güth and Yaari (1992) and Leininger (2009) among others—we equate fitness with expected material payoffs. A player's preferences (expected utility) differ from fitness (expected material payoff) to the extent that a player's perceived prize valuation differs from the prize's material value. Equilibrium behavior characterized by (3) induces fitness given by

$$\begin{aligned} F_i(v_1, v_2) &= \frac{x_i^*(v_1, v_2)}{x_1^*(v_1, v_2) + x_2^*(v_1, v_2)} v - x_i^*(v_1, v_2) \\ &= \frac{v_i^2 v + v_i v_j v - v_i^2 v_j}{(v_1 + v_2)^2} \end{aligned} \quad (4)$$

for  $i, j = 1, 2, i \neq j$ .

Suppose, without loss of generality, that player 2 is a member of a finite population endowed with the evolutionarily stable preferences (ESP)  $v^*$  and that player 1 is a single mutant with preferences  $v_1$ . A single mutant will *always* match with a member of the population with  $v^*$  and a member of the population with  $v^*$  will match with the mutant with probability  $1/(N - 1)$ . Therefore, the fitness of player 1 relative to a member of the population with  $v^*$  is given by

$$R_1(v_1, v^*) = F_1(v_1, v^*) - \left( \frac{1}{N - 1} F_2(v_1, v^*) + \frac{N - 2}{N - 1} F_2(v^*, v^*) \right). \quad (5)$$

We follow Eaton and Eswaran (2003) and Leininger (2009) in adapting the definition of finite population evolutionary stability from Schaffer (1988) and define finite population ESP  $v^*$  as

$$v^* \in \arg \max_{v_1} R_1(v_1, v^*). \quad (6)$$

The following proposition characterizes ESP for finite populations.

**Proposition 1.** *For a finite population interacting in pairwise contests, the unique evolutionarily stable preferences  $v^*$  overstate the material value of the prize. Namely,*

$$v^* = \frac{N}{N - 1} v > v. \quad (7)$$

*Proof.* For a finite population, the first-order condition for the problem in (6) is

$$\frac{v_1 v^* v + v^{*2} v - 2v_1 v^{*2}}{(v_1 + v^*)^3} - \frac{1}{N - 1} \frac{-v_1 v^* v + v_1 v^{*2} - v^{*2} v - v^{*3}}{(v_1 + v^*)^3} = 0. \quad (8)$$

Setting  $v_1 = v^*$  in (8) and solving for  $v^*$  yields (7).

To establish this unique interior solution as the unique global maximizer of relative fitness in (5), note that the second-order condition is given by

$$\frac{-2v_1v^*v - 2v^{*2}v + 4v_1v^{*2} - 2v^{*3}}{(v_1 + v^*)^4} - \frac{1}{N-1} \frac{2v_1v^*v + 2v^{*2}v - 2v_1v^{*2} + 4v^{*3}}{(v_1 + v^*)^4} < 0. \quad (9)$$

Setting  $v_1 = v^* = Nv/(N-1)$ , substituting into (9), and simplifying reveals that the second-order condition holds if and only if

$$-Nv - \frac{N}{N-1}v < 0,$$

and thus the second-order condition holds at the interior solution. Substituting  $v^* = Nv/(N-1)$  into (9) and simplifying reveals that the second-order condition holds if and only if

$$v_1 - \frac{N+3-2/N}{N-1}v < 0$$

so that relative fitness in (5) is concave for  $v_1 \in (0, \tilde{v})$  and convex for  $v_1 \in (\tilde{v}, \infty)$ , where

$$\tilde{v} = \frac{N+3-2/N}{N-1}v > \frac{N}{N-1}v = v^*.$$

Comparing the relative fitness of the interior solution  $v^* = Nv/(N-1)$  to the relative fitness of the corner solution 0, we have  $R_1(v^*, v^*) = 0$  and  $R_1(0, v^*) = -N^2/4(N-1)^2 < 0$ , respectively. Thus, (7) is the unique global maximizer of (5).  $\square$

Relative fitness maximization provides the intuition for why finite population ESP overstate the material value of the prize. Preferences overstating the material value of the prize commit a player to making a relatively larger expenditure in the contest than a player whose preferences match exactly the material value of the prize. The increased contest expenditure creates a second-order loss in own fitness but a first-order loss in the rival player's fitness through the negative externality of expenditures in the contest success function in (1); together, these effects constitute an increase in relative fitness and thus an evolutionary advantage for preferences overstating the material value of the prize.

The above intuition for finite population ESP does *not* hold for infinite populations; for infinite populations, evolutionary selection is based upon absolute fitness. Analogous to the case of finite populations, we adapt the definition of infinite population evolutionary stability from Maynard Smith and Price (1973) and define infinite population ESP as the  $v^{**}$  for which (i)  $F_1(v^{**}, v^{**}) \geq F_1(v_1, v^{**})$  for all  $v_1$  and (ii)  $F_1(v^{**}, v^{**}) > F_1(\hat{v}, \hat{v})$  for all  $\hat{v}$

such that  $F_1(\hat{v}, v^{**}) = F_1(v^{**}, v^{**})$ ; symmetric conditions hold for player 2. The following proposition characterizes ESP for infinite populations.

**Proposition 2.** *For an infinite population interacting in pairwise contests, the unique evolutionarily stable preferences  $v^{**}$  match exactly the material value of the prize so that*

$$v^{**} = v. \quad (10)$$

*Proof.* Taking the perspective of player 1, infinite population ESP solve

$$\max_{v_1} F_1(v_1, v^{**}) = \frac{v_1^2 v + v_1 v^{**} v - v_1^2 v^{**}}{(v_1 + v^{**})^2}; \quad (11)$$

evolution solves a symmetric problem for player 2, so taking the perspective of player 1 is without loss of generality. The first-order condition for the problem in (11) is

$$\frac{v_1 v^{**} v + v^{**2} v - 2v_1 v^{**2}}{(v_1 + v^{**})^3} = 0. \quad (12)$$

Setting  $v_1 = v^{**}$  in (12) and simplifying yields

$$\frac{v - v^{**}}{4v^{**}} = 0, \quad (13)$$

which is satisfied by  $v^{**} = v$  as in (10).

To establish this unique interior solution as the unique global maximizer of (11), note that the second-order condition is given by

$$\frac{-2v_1 v^{**} v - 2v^{**2} v + 4v_1 v^{**2} - 2v^{**3}}{(v_1 + v^{**})^4} < 0. \quad (14)$$

Setting  $v_1 = v^{**} = v$ , substituting into (14), and simplifying reveals that the second-order condition holds if and only if

$$-\frac{1}{8v} < 0,$$

and thus the second-order condition holds at the interior solution. Substituting  $v^{**} = v$  into (14) and simplifying reveals that the second-order condition holds if and only if

$$v_1 - 2v < 0$$

so that absolute fitness in (11) is concave for  $v_1 \in (0, \tilde{v})$  and convex for  $v_1 \in (\tilde{v}, \infty)$ , where

$$\tilde{v} = 2v > v = v^{**}.$$

Comparing the relative fitness of the interior solution  $v^{**} = v$  to the absolute fitness of the corner solution 0, we have  $F_1(v^{**}, v^{**}) = v/4 > 0$  and  $F_1(0, v^{**}) = 0$ , respectively. Thus, (10) is the unique global maximizer of (11).  $\square$

Again, considering absolute fitness maximization as the target of evolution in infinite populations provides the intuition for why infinite population ESP match exactly the material value of the prize. A mutant is measure zero in an infinite population, so any mutant whose preferences understate (overstate) the material value of the prize imposes only a loss on own fitness from its decreased (increased) expenditure. Therefore, preferences that do not match exactly the material value of the prize are at an evolutionary disadvantage.

A tight link exists between the behavior ESP induce and the strategies evolution would select were it to act directly upon behavior (evolutionarily stable strategies, ESS) in pairwise contests. From (3), finite population ESP induce the expenditure behavior

$$x^* \left( \frac{N}{N-1}v, \frac{N}{N-1}v \right) = \frac{N}{4(N-1)}v$$

and infinite population ESP induce the expenditure behavior

$$x^*(v, v) = \frac{v}{4}.$$

From Hehenkamp et al. (2004), it is well-known that the finite population ESS for pairwise contests is

$$x_N^{ESS} = \frac{N}{4(N-1)}v$$

and that the infinite population ESS for pairwise contests is

$$x_\infty^{ESS} = \frac{v}{4}.$$

We record this equivalence in the following corollary.

**Corollary 1.** *Evolutionarily stable preferences from indirect evolution in pairwise contests induce behavior that is equivalent to the evolutionarily stable strategies from direct evolution in pairwise contests.*

Leininger (2009) obtains an analogous equivalence result for the evolution of interde-



pendent preferences (namely, the weight a player's preferences place upon a rival player's expected material payoff). Our result suggests that the equivalence between the behavior ESP induce and ESS in pairwise contests is robust. The result is also a complement to Leininger (2009) since we establish that this equivalence holds for a class of *independent* preferences where evolution acts upon players' perceived prize valuations.

### 3 Conclusion

If evolution acts upon the perceived prize valuations of members of a finite population interacting in pairwise contests, then preferences will evolve to overstate the material value of the prize. These evolutionarily stable preferences thus provide a qualitative rationale for the overexpenditure observed in experimental contests. If, instead, evolution works upon the preferences of members from an infinite population, evolutionarily stable preferences will evolve to match exactly the prize's material value. The difference in these results arises from a mutant's ability (inability) to improve relative fitness in finite (infinite) populations by perceiving the value of winning a contest as more than the material value of the prize. Should evolution instead work directly upon strategies, then the evolutionarily stable strategies are equivalent to the behavior evolutionarily stable preferences induce. We leave for future research the determination of general conditions under which the behavior evolutionarily stable preferences induce is equivalent to evolutionarily stable strategies in contests.

### Acknowledgements

We thank Kyung Hwan Baik and Hartmut Kliemt for their helpful comments and suggestions.

### References

- Baik, K. H. (1994). Effort Levels in Contests with Two Asymmetric Players. *Southern Economic Journal*, 61(2), 367–378.
- Baye, M. R., & Hoppe, H. C. (2003). The Strategic Equivalence of Rent-Seeking, Innovation, and Patent-Race Games. *Games and Economic Behavior*, 44(2), 217–226.
- Baye, M. R., Kovenock, D., & de Vries, C. G. (1993). Rigging the Lobbying Process: An Application of the All-Pay Auction. *American Economic Review*, 83(1), 289–294.

- Baye, M. R., Kovenock, D., & de Vries, C. G. (1996). The All-Pay Auction with Complete Information. *Economic Theory*, 8(2), 291–305.
- Baye, M. R., Kovenock, D., & de Vries, C. G. (2005). Comparative Analysis of Litigation Systems: An Auction-Theoretic Approach. *Economic Journal*, 115(505), 583–601.
- Corchón, L. C. (2007). The Theory of Contests: A Survey. *Review of Economic Design*, 11(2), 69–100.
- Eaton, B., & Eswaran, M. (2003). The Evolution of Preferences and Competition: A Rationalization of Veblen’s Theory of Invidious Comparisons. *Canadian Journal of Economics*, 36(4), 832–859.
- Fang, H. (2002). Lottery Versus All-Pay Auction Models of Lobbying. *Public Choice*, 112(3–4), 351–371.
- Farmer, A., & Pecorino, P. (1999). Legal Expenditure as a Rent-Seeking Game. *Public Choice*, 100(3–4), 271–288.
- Garfinkel, M. R., & Skaperdas, S. (2007). Economics of Conflict: An Overview. In Sandler, T., & Hartley, K. (Eds.), *Handbook of Defense Economics, Volume 2: Defense in a Globalized World*, pp. 649–709. North-Holland, Amsterdam.
- Goeree, J. K., Maasland, E., Onderstal, S., & Turner, J. L. (2005). How (Not) to Raise Money. *Journal of Political Economy*, 113(4), 897–918.
- Güth, W., & Yaari, M. E. (1992). Explaining Reciprocal Behavior in Simple Strategic Games: An Evolutionary Approach. In Witt, U. (Ed.), *Explaining Process and Change: Approaches to Evolutionary Economics*, pp. 23–34. University of Michigan Press, Ann Arbor.
- Hehenkamp, B., Leininger, W., & Possajennikov, A. (2004). Evolutionary Equilibrium in Tullock Contests: Spite and Overdissipation. *European Journal of Political Economy*, 20(4), 1045–1057.
- Herrmann, B., & Orzen, H. (2008). The Appearance of Homo Rivalis: Social Preferences and the Nature of Rent Seeking. CeDEx Discussion Paper 2008-10, University of Nottingham.
- Hillman, A. L., & Riley, J. G. (1989). Politically Contestable Rents and Transfers. *Economics & Politics*, 1(1), 17–39.

- Hirshleifer, J. (1995). Theorizing About Conflict. In Hartley, K., & Sandler, T. (Eds.), *Handbook of Defense Economics, Volume 1*, pp. 165–189. North-Holland, Amsterdam.
- Hörisch, H., & Kirchkamp, O. (2010). Less Fighting than Expected: Experiments with Wars of Attrition and All-Pay Auctions. *Public Choice*, 144(1-2), 347–367.
- Konrad, K. A. (2004). Altruism and Envy in Contests: An Evolutionarily Stable Symbiosis. *Social Choice and Welfare*, 22(3), 479–490.
- Kooreman, P., & Schoonbeek, L. (1997). The Specification of the Probability Functions in Tullock’s Rent-Seeking Contest. *Economics Letters*, 56(1), 59–61.
- Leininger, W. (2003). On Evolutionarily Stable Behavior in Contests. *Economics of Governance*, 4(3), 177–186.
- Leininger, W. (2009). Evolutionarily Stable Preferences in Contests. *Public Choice*, 140(3-4), 341–356.
- Maynard Smith, J., & Price, G. R. (1973). The Logic of Animal Conflict. *Nature*, 246(5427), 15–18.
- Mohlin, E. (2010). Internalized Social Norms in Conflicts: An Evolutionary Approach. *Economics of Governance*, 11(2), 169–181.
- Morgan, J. (2000). Financing Public Goods by Means of Lotteries. *Review of Economic Studies*, 67(4), 761–784.
- Morgan, J., Orzen, H., & Sefton, M. (2010). Endogenous Entry in Contests. *Economic Theory*, forthcoming.
- Nitzan, S. (1994). Modelling Rent-Seeking Contests. *European Journal of Political Economy*, 10(1), 41–60.
- Nti, K. O. (1999). Rent-Seeking with Asymmetric Valuations. *Public Choice*, 98(3-4), 415–430.
- Öncüler, A., & Croson, R. (2005). Rent-Seeking for a Risky Rent: A Model and Experimental Investigation. *Journal of Theoretical Politics*, 17(4), 403–429.
- Potters, J., de Vries, C. G., & van Winden, F. (1998). An Experimental Examination of Rational Rent-Seeking. *European Journal of Political Economy*, 14(4), 783–800.

- Schaffer, M. E. (1988). Evolutionarily Stable Strategies for a Finite Population and a Variable Contest Size. *Journal of Theoretical Biology*, 132(4), 469–478.
- Schmidt, F. (2009). Evolutionary Stability of Altruism and Envy in Tullock Contests. *Economic Governance*, 10(3), 247–259.
- Sheremeta, R. M. (2010). Experimental Comparison of Multi-Stage and One-Stage Contests. *Games and Economic Behavior*, 68(2), 731–747.
- Skaperdas, S. (1996). Contest Success Functions. *Economic Theory*, 7(2), 283–280.
- Stein, W. E. (2002). Asymmetric Rent-Seeking with More than Two Contestants. *Public Choice*, 113(3-4), 325–336.
- Szymanski, S. (2003). The Economic Design of Sporting Contests. *Journal of Economic Literature*, 41(4), 1137–1187.
- Tullock, G. (1980). Efficient Rent Seeking. In Buchanan, J., Tullock, G., & Tollison, R. (Eds.), *Toward a Theory of the Rent-Seeking Society*, pp. 97–112. Texas A&M University Press.
- Wärneryd, K. (2002). Rent, Risk, and Replication: Preference Adaptation in Winner-Take-All Markets. *Games and Economic Behavior*, 41(2), 344–364.